Areas and Volumes
Module 1

Module 1 Description:

Middle school teachers examine how concepts involving areas and volumes progress from sixth grade to calculus. Training begins with manipulative-rich student lessons that explore the surface area and volume of three-dimensional solids. As the lessons progress through the vertical strand, participants will come to realize the necessity of teaching both area and volume on the coordinate plane. They work selected questions from and discuss teaching strategies for model lessons for middle grades in which students plot coordinate points to create planar figures, calculate the areas of those figures, revolve the figures about a horizontal or vertical line to create three-dimensional solids, and calculate the volumes of those solids. In addition, they will work through and discuss lessons from Algebra 1 and Geometry or Math 1 and Math 2 in which students begin by graphing linear equations to create these models.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topics of areas and volumes to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
    - describe the effect of scale changes on surface area and volume.
    - represent three-dimensional figures with nets and use the nets to calculate surface area.
    - model three-dimensional solids formed when various regions are revolved about horizontal or vertical lines.
    - calculate the surface area and volume of solids of revolution.
- identify instructional strategies that they can use to assist students in developing the habits of mind that are required for college and career readiness.

Rate of Change: Average and Instantaneous
Module 2

Module 2 Description:

Middle school teachers examine how concepts involving rate of change progress from sixth grade to calculus. Training begins with a manipulative-rich lesson that leads students to interpret pi as a constant rate of change. As the lessons progress through the vertical strand, participants connect slope and rate of change. They work selected questions from and discuss teaching strategies for model lessons for middle grades in which students interpret speed as a rate of change, calculate average rates of change over specific intervals, and examine graphs to conclude that average rate of change is not the same as instantaneous rate of change. In addition, teachers work through and discuss lessons from Algebra 1 and Geometry or Math 1 and Math 2 in which students apply a difference quotient to calculate average rate of change and are introduced to approximating an instantaneous rate of change from a graph.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of rate of change to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
  - identify a constant rate of change from a table or graph.
  - complete a table or sketch a graph using a constant rate of change.
  - model rate of change using exploratory activities, role play, and motion detectors.
  - differentiate between average and instantaneous rates of change of a function.
  - calculate average rate of change using a difference quotient.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Graphical Displays and Distributions
Module 3

Module 3 Description:

Middle school teachers examine how concepts involving graphical displays and distributions progress from sixth grade to statistics. Participants examine classroom-ready middle grades lessons which require students to construct, compare, analyze, and interpret box-and-whisker plots, line plots (dotplots), histograms, and stem-and-leaf plots. Each lesson involves creating, by hand or with a graphing calculator, appropriate graphical displays based on real-world data. Graphs are analyzed by investigating measures of central tendency, variability, and shape. In addition, teachers work through and discuss a lesson for Geometry or Math 2 in which students observe variability in data collected using measurement. Algebra 1 or Math 1 lessons connecting transformations to graphical displays and introducing standard deviation as a measure of variability are also explored in the training.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topics of graphical displays and distributions to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level statistics in an AP class or university setting.
    - construct, compare, analyze, and interpret histograms, box-and-whisker plots, line plots (dotplots), and stem-and-leaf plots.
    - analyze graphs using measures of central tendency, variability, and shape.
    - describe distributions using mean, mean absolute deviation, and standard deviation.
    - identify the effects of measurement inaccuracies on the distribution of area and volume calculations.
    - construct box-and-whisker plots of randomly selected function values over a limited domain and analyze the effects of transformations on the distribution of those function values.
    - connect graphical displays of randomly selected function values over a limited domain to the behavior of the function over that limited domain.
    - demonstrate graphing calculator skills used in creating graphical displays and calculating distributions.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Analysis of Functions: Piecewise Graphs
Module 4

Module 4 Description:

Middle school teachers examine how concepts involving analysis of functions progress from sixth grade to calculus. Training begins with a strategy for teaching students the difference between the path of a person's walk and a distance-time graph of the walk. Participants work through and discuss teaching strategies for classroom-ready middle grades lessons and assessments in which students interpret distance-time and rate-time graphs. In addition, they discuss selected questions from Algebra 1 and Geometry or Math 1 and Math 2 lessons on graphing and defining piecewise functions.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of analysis of piecewise functions to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
  - interpret distance-time and rate-time graphs.
  - model distance-time and rate-time graphs with role play.
  - write equations of piecewise functions.
  - analyze key features of piecewise functions.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Accumulation
Module 5

Module 5 Description:

Middle school teachers explore how concepts involving accumulating area progress from sixth grade to calculus. Participants work through and discuss teaching strategies for manipulative-rich middle grades lessons in which students approximate the areas of irregular shapes using a variety of techniques. As the lessons progress through the vertical strand, participants explore the implications of accumulating the area under a rate graph and consider area calculations where the dimension of the answer is not square units. In addition, they discuss selected questions from Algebra 1 and Geometry or Math 1 and Math 2 lessons that apply accumulation to graphs of functions, using left- and right-hand rectangles and trapezoids.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of accumulation to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
    - estimate areas of irregular figures drawn on grid paper and recognize that smaller grids lead to more accurate estimates.
    - estimate areas of irregular figures by drawing in and measuring the dimensions of rectangles or trapezoids.
    - analyze the units that result from calculating an area enclosed by a rate graph.
    - estimate the area between a curve and the x-axis using left-hand and right-hand rectangles.
    - estimate the area between a curve and the x-axis using trapezoids and connect the trapezoidal approximation to the mean of the left-hand and right-hand rectangle approximations.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Probability
Module 6

Module 6 Description:

Middle school teachers examine how concepts involving probability progress from sixth grade to statistics. Participants work through and discuss teaching strategies for classroom-ready middle grades lessons that require students to interpret Venn diagrams, two-way tables, and sample spaces as they calculate simple and conditional probabilities. Teachers also investigate a lesson in which students calculate probabilities using geometric shapes and a lesson applying Pascal's Triangle to binomial probabilities. In addition, teachers will discuss selected questions from Algebra 1 and Geometry or Math 1 and Math 2 lessons in which students calculate probabilities by conducting simulations and interpreting tree diagrams.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of probability to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level statistics in an AP class or university setting.
  - interpret Venn diagrams, two-way tables, tree diagrams, and sample spaces to calculate probabilities.
  - differentiate between experimental and theoretical probability.
  - conduct simulations and collect data.
  - apply Pascal's Triangle to binomial probability.
  - calculate probability using geometric shapes.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Position/Velocity/Acceleration
Module 7

Module 7 Description:

Middle school teachers examine how concepts involving position, velocity, and acceleration progress from sixth grade to calculus. Training begins with a motion detector activity designed to explore motion along a horizontal line. Participants work through and discuss teaching strategies for middle grades lessons that require students to connect a line graph of the path of motion with a coordinate graph of distance versus time, sketch a graph from a story, and interpret graphs from verbal descriptions. In addition, they will discuss selected questions from Algebra 1 and Geometry or Math 1 and Math 2 lessons in which students analyze and compare graphs of position, velocity, and acceleration.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of position/velocity/acceleration to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
  - create position graphs using exploratory activities, role play, and motion detectors.
  - connect line graphs of horizontal motion with coordinate graphs of distance versus time using verbal descriptions of the motion.
  - describe transformational changes to a position graph in terms of the situation.
  - analyze distance and speed graphs.
  - understand the relationship between position and velocity graphs.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Analysis of Functions: Transformations
Module 8

Module 8 Description:

Middle school teachers examine how concepts involving transformations progress from sixth grade to calculus. Training begins with examining a manipulative-rich activity in which students create their own tessellations using translations and rotations. As the lessons progress through the vertical strand, participants work through and discuss teaching strategies for middle grades lessons in which students explore transformations applied to graphs on the coordinate plane. In addition, they discuss selected questions from Algebra 1 and Geometry or Math 1 and Math 2 lessons in which students apply their understanding of transformations to functions, including the impact on domain and range, and engage in a fun game of transformation charades.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of transformations to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
  - apply transformations to pictures and graphs.
  - identify the impact of transformations on coordinate points.
  - describe the effects of transformations physically, verbally, analytically, numerically, and graphically.
  - interpret the meaning of transformations in the context of a real-world situation.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Bivariate Data
Module 9

Module 9 Description:

Middle school teachers examine how concepts involving analyzing bivariate data progress from sixth grade to calculus. Training begins with an engaging activity that introduces many of the skills involved in collecting, plotting, and analyzing bivariate data. Teachers work through and discuss teaching strategies for classroom-ready lessons in which students investigate data collection and coding, fitting functions to data, and using models to predict additional values. In addition, they discuss selected questions from Algebra 1 and Geometry or Math 1 and Math 2 lessons in which students interpret the slope and intercepts of a linear model in the context of the question and are introduced to the concept of a residual.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of analyzing bivariate data to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level statistics in an AP class or university setting.
  - collect, code, and plot real-world data.
  - fit linear functions to data, use the models to predict other values, and interpret the meanings of slope and intercepts in the context of the situation.
  - understand a residual as a means to analyze the fit of a regression model.
  - demonstrate graphing calculator skills used in analyzing bivariate data.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Optimization: Area and Volume Applications
Module 10

Module 10 Description:

Middle school teachers examine how concepts involving optimization progress from sixth grade to calculus. Training begins with an engaging collaborative activity in which participants build boxes with a variety of dimensions and use multiple strategies to judge which box has the maximum volume. Participants work through and discuss teaching strategies for hands-on lessons that require students to measure and model in optimization situations in order to formulate generalizations about optimum areas for fixed perimeters. In addition, teachers discuss selected questions from Algebra 1, Geometry, Math 1 and Math 2 lessons in which students apply a process for modeling and solving real-world optimization situations.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of optimization of areas and volumes to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
  - approximate optimal solutions using measurement and models.
  - formulate generalizations about optimum areas for fixed perimeters.
  - write functions to model real-world applications of area and volume, identify reasonable domains and ranges, and then determine optimum solutions.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Rate of Change: Related Rates
Module 11

Module 11 Description:

Middle school teachers examine how concepts involving related rates of change progress from sixth grade to calculus. Training begins with a review of rewriting geometric formulas and other literal equations in terms of specific quantities. As the training progresses through the vertical strand, participants work through and discuss teaching strategies for middle grades lessons that explore dynamic situations where a change in one quantity results in a change in another related quantity. In addition, the "Fill It Up" series of lessons allows teachers to observe how concepts in the Algebra 1, Geometry, and Algebra 2 or Math 1, Math 2 and Math 3 lessons are built on ideas that are introduced at the middle grades level.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of related rates of change to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
    - rewrite geometric formulas and literal equations in terms of specific quantities.
    - model dynamic situations where a change in one quantity causes a related change in another quantity and recognize that the rates at which those quantities change are linked through their mathematical relationship.
    - solve real-life applications of situations that involve related rates of change.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Analysis of Functions: Curve Sketching
Module 12

Module 12 Description:

Middle school teachers examine how concepts involving curve sketching progress from sixth grade to calculus. Teachers will delve into student lessons involving sketching functions with given characteristics based on information provided verbally, numerically, and analytically. They work selected questions from and discuss teaching strategies for model lessons for middle grades in which students match scenarios to distance-time graphs and explore key features of linear and non-linear graphs. In addition, they will work through and discuss Algebra 1/Math 1 lessons in which students interpret verbal descriptions of the domain, range, and specific characteristics of a piecewise function to graph the function and to write linear and quadratic equations for the separate pieces of the function.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of limits to increase alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for high school math courses leading to college-level calculus in an AP class or university setting.
  - o sketch functions with given characteristics based on information provided verbally, numerically, and analytically.
  - o match distance-time graphs to scenarios in real-world contexts.
  - o analyze characteristics of graphs.
  - o practice matching function equations, descriptions, tables, and graphs.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.