Areas and Volumes
Module 1

Module 1 Description:

High school teachers examine how concepts involving areas and volume progress from sixth grade to calculus. Participants begin by reviewing how these concepts are introduced at the middle school level. As the lessons progress through the vertical strand, participants come to realize the necessity of teaching both area and volume on the coordinate plane. They work selected questions from and discuss teaching strategies for high school model lessons. In these activities, teachers experience how students graph linear equations and/or inequalities to create planar figures, calculate the areas of those figures, revolve the figures about a horizontal or vertical line to create three-dimensional solids, and calculate the volumes of those solids. In addition, teachers engage in an activity to explore maximizing the volume of a cone.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topics of areas and volumes to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level calculus in an AP class or university setting.
  - express area and volume formulas in terms of specific variables.
  - model three-dimensional solids formed when various regions are revolved about horizontal or vertical lines.
  - calculate the surface area and volume of solids of revolution.
- Identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Rate of Change: Average and Instantaneous
Module 2

Module 2 Description:

High school teachers examine how concepts involving rate of change progress from sixth grade to calculus. Participants begin by reviewing how these concepts are introduced at the middle school level. As the lessons progress through the vertical strand, participants differentiate between average and instantaneous rates of change. They work selected questions from and discuss teaching strategies for high school model lessons in which students apply a difference quotient to calculate average rates of change over specific intervals, approximate an instantaneous rate of change, and calculate slopes of curves that are not functions. In addition, participants model piecewise distance graphs with a motion detector and graph the corresponding speed graphs.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of rate of change to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level calculus in an AP class or university setting.
  - differentiate between average and instantaneous rates of change of a function.
  - calculate average rate of change using a difference quotient.
  - model rate of change using exploratory activities, role play, and motion detectors.
  - estimate instantaneous rate of change using the slopes of secant lines to approach the slope of a tangent line.
  - apply a formula for instantaneous rate of change at a point on a curve to identify characteristics of the curve.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Graphical Displays and Distributions
Module 3

Module 3 Description:

High school teachers examine how concepts involving graphical displays and distributions progress from sixth grade to statistics. Training begins with an introductory activity that provides a comprehensive overview of all of the skills included in the middle grades lessons. Teachers work through and discuss classroom-ready lessons in which students observe variability in data collected using measurement, connect transformations to graphical displays, compute standard deviation, and use the Empirical rule and z-scores to estimate population percentages for normal distributions.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topics of graphical displays and distributions to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level statistics in an AP class or university setting.
    - construct, compare, analyze, and interpret histograms, box-and-whisker plots, line plots (dotplots), and stem-and-leaf plots.
    - analyze graphs using measures of central tendency, variability, and shape.
    - describe distributions using mean and standard deviation.
    - apply the Empirical Rule and z-scores to estimate population percentages for normal distributions.
    - identify the effects of measurement inaccuracies on the distribution of area and volume calculations.
    - construct box-and-whisker plots of randomly selected function values over a limited domain and analyze the effects of transformations on the distribution of those function values.
    - connect graphical displays of randomly selected function values over a limited domain to the behavior of the function over that limited domain.
    - demonstrate graphing calculator skills used in creating graphical displays and calculating distributions.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Analysis of Functions: Piecewise Graphs
Module 4

Module 4 Description:

High school teachers examine how concepts involving analysis of functions progress from sixth grade to calculus. Participants begin by reviewing how these concepts are introduced at the middle school level using distance-time and rate-time graphs. As the training progresses through the vertical strand, teachers work selected questions from and discuss teaching strategies for the high school lessons. In these activities, students write equations for, identify key features of, and analyze the effects of transformations on piecewise graphs.

Learner Outcomes:

Participants will

- compare expectations of students from sixth grade math through pre-calculus on the topic of analysis of piecewise functions to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level calculus in an AP class or university setting.
  - interpret distance-time and rate-time graphs.
  - model distance-time and rate-time graphs with role play.
  - write equations of piecewise functions.
  - analyze key features of piecewise functions.
  - apply transformations on specific portions of a piecewise function to make the function continuous.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Accumulation
Module 5

Module 5 Description:

High school teachers explore how concepts involving accumulating area progress from sixth grade to calculus. Participants begin by examining how these concepts are introduced at the middle school level. As the lessons progress through the vertical strand, participants consider area calculations where the dimension of the answer is not square units and explore the implications of accumulating the area under a rate graph. They work through and discuss teaching strategies for classroom-ready high school lessons and assessments in which students approximate the areas of regions bounded by functions, using left-hand, right-hand, and midpoint rectangles, as well as trapezoids. In addition, they examine real-world situations represented by rate functions and apply accumulation to answer questions about the situation.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of accumulation to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level calculus in an AP class or university setting.
  - analyze the units that result from calculating an area enclosed by a rate graph.
  - estimate the area between a curve and the x-axis using left-hand, right-hand, and midpoint rectangles.
  - estimate the area between a curve and the x-axis using trapezoids and connect the trapezoidal approximation to the mean of the left-hand and right-hand rectangle approximations.
  - apply accumulation to real-world rate functions to answer questions in the context of the situation.
  - conclude that the accuracy of the area approximation is improved by increasing the number of subdivisions of the bounded area.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Probability
Module 6

Module 6 Description:

High school teachers examine how concepts involving probability progress from sixth grade to statistics. Participants work through and discuss teaching strategies for classroom-ready high school lessons that require students to interpret sample spaces, two-way tables, tree diagrams, and general probability rules as they calculate simple, conditional, and compound probabilities. Teachers also investigate lessons in which students employ Pascal's Triangle, the Binomial Theorem, permutations, and combinations in probability calculations. In addition, teachers will conduct simulations to approximate probabilities.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of probability to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level statistics in an AP class or university setting.
  - interpret sample spaces, two-way tables, tree diagrams, and geometric figures to calculate probabilities.
  - differentiate between experimental and theoretical probability.
  - conduct simulations and collect data.
  - apply Pascal's Triangle, the Binomial Theorem, permutations, and combinations to probability calculations.
  - use general probability rules to calculate compound and conditional probabilities and to determine whether or not two events are independent.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Position/Velocity/Acceleration
Module 7

Module 7 Description:

High school teachers examine how concepts involving position, velocity, and acceleration progress from sixth grade to calculus. Training begins with a motion detector activity designed to explore motion along a horizontal line. Participants work through and discuss teaching strategies for high school lessons that require students to connect a line graph of the path of motion with a coordinate graph of distance versus time. In addition, teachers explore lessons that require students to analyze and compare graphs of position, velocity, and acceleration, to interpret motion graphs presented functionally and parametrically, and to apply exponential and logarithmic functions to position, velocity, and acceleration concepts.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of position/velocity/acceleration to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level calculus in an AP class or university setting.
  - model position, velocity, and acceleration involved in motion along a horizontal line using motion detectors.
  - connect line graphs of horizontal motion with coordinate graphs of distance versus time using verbal descriptions of the motion.
  - interpret and relate position, velocity, and acceleration graphs.
  - compare particle motion graphed functionally and parametrically.
  - differentiate between velocity and speed.
  - analyze position, velocity and acceleration represented with exponential and logarithmic functions.
  - demonstrate graphing calculator skills used in position/velocity/acceleration.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Analysis of Functions: Transformations
Module 8

Module 8 Description:

High school teachers examine how concepts involving transformations of functions progress from sixth grade to calculus. Participants begin by reviewing how these concepts are introduced at the middle school level. As the lessons progress through the vertical strand, participants work through and discuss teaching strategies for high school lessons that require students to recognize transformations presented in a variety of modalities and to connect transformations with their impact on function notation, on graphs, and on domain and range. Teachers also investigate lessons in which students interpret transformations in terms of real-world situations and actively engage in a fun game of transformation charades.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of transformations to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level statistics in an AP class or university setting.
    - connect transformations to the point-slope form of a linear equation and to the transformational form of other parent functions as strategies for graphing.
    - identify transformations presented in physical, verbal, analytical, numerical, and graphical modalities.
    - evaluate the impact of transformations on function values and key features of function graphs.
    - interpret the meaning of transformations in the context of a real-world situation.
    - demonstrate graphing calculator skills used in transformations.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Bivariate Data
Module 9

Module 9 Description:

High school teachers examine how concepts involving linear and non-linear bivariate data progress from sixth grade to calculus. Training begins with an engaging activity that introduces many of the skills involved in collecting, plotting, and analyzing bivariate data. Teachers work through and discuss teaching strategies for classroom-ready lessons in which students investigate data collection and coding, fitting functions to data, interpreting the meanings of the parameters in the functions, and using the models to predict additional values. Teachers also investigate lessons that introduce students to techniques for straightening curved data in order to identify models of best fit and to using residuals to analyze regression models.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of analyzing bivariate data to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level statistics in an AP class or university setting.
    - collect and use real-world data to develop the skills of coding and plotting data, fitting functions to data, using the models to predict other values, and interpreting the meanings of the parameters in the models in the context of the situation.
    - analyze regression models using residuals.
    - use techniques for straightening curved data to identify models of best fit.
    - demonstrate graphing calculator skills used in analyzing bivariate data.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Optimization: Area and Volume Applications
Module 10

Module 10 Description:

High school teachers examine how concepts optimization progress from sixth grade to calculus. Participants begin by exploring a hands-on activity involving measurement and data collection to locate a maximum area. As the lessons progress through the vertical strand, teachers work through and discuss teaching strategies for a wide variety of real-world situations where maximizing or minimizing area and volume is the goal. Participants engage in a highly collaborative activity in which they build boxes with a variety of dimensions, informally judge the order of the boxes by volume, and then use visual, numerical, graphical, and functional representations of the data to locate the optimum solution.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of optimization of areas and volumes to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level calculus in an AP class or university setting.
  - approximate optimal solutions using measurement and models.
  - determine solution strategies for approaching optimization questions.
  - write functions for area and volume from descriptions of real-world applications, determine reasonable domains and ranges, and then determine optimum solutions.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Rate of Change: Related Rates
Module 11

Module 11 Description:

High school teachers examine how concepts involving related rates of change progress from sixth grade to calculus. Training begins with a review of rewriting geometric formulas and other literal equations in terms of specific quantities. As the training progresses through the vertical strand, participants will work through and discuss teaching strategies for high school lessons that explore dynamic situations where a change in one quantity results in a change in another related quantity. Teachers experience the ways in which students engage in hands-on activities that physically demonstrate the varying rates of change and then determine the mathematical relationships that influence the rates, including real-world situations that apply triangles, curves, areas, and volumes to the concepts of related rates.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of related rates of change to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level calculus in an AP class or university setting.
  - solve literal equations.
  - model dynamic situations where a change in one quantity causes a related change in another quantity, and recognize that the rates at which those quantities change are linked through their mathematical relationship
  - solve real-life applications questions for related rates situations.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.

Analysis of Functions: Curve Sketching
Module 12

Module 12 Description:

High school teachers examine how concepts involving curve sketching progress from sixth grade to calculus. Teachers will delve into student lessons involving sketching functions with given characteristics based on information provided verbally, numerically, and analytically. After reviewing how these concepts are introduced at the middle school level, participants work selected questions from and discuss teaching strategies for high school model lessons. In these activities, teachers experience how students explore key features of linear and non-linear graphs and interpret verbal descriptions of specific characteristics of piecewise functions to graph and write equations for these functions. In addition, participants are introduced to games that can be used in the classroom to enhance and further develop understanding of the vocabulary used in analyzing functions and to make connections between multiple modalities in transformations of parent functions.

Learner Outcomes:

Participants will

- compare expectations for students from sixth grade math through pre-calculus on the topic of transformations to increase vertical alignment.
- apply deeper content-based knowledge to increase instructional rigor in order to prepare students for college-level statistics in an AP class or university setting.
  - sketch functions with given characteristics based on information provided verbally, numerically, and analytically.
  - analyze characteristics of graphs.
  - practice matching function equations, descriptions, tables, and graphs.
  - graph rational functions.
- identify instructional strategies that teachers can use to assist students in developing the habits of mind that are required for college and career readiness.