



**LEVEL**

Grade 8, Algebra 1, or Math 1 in a unit with rate of change and slope

**MODULE/CONNECTION TO AP\***

Rate of Change

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**MODALITY**

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.



P – Physical  
V – Verbal  
A – Analytical  
N – Numerical  
G – Graphical

# Slope Investigation

**ABOUT THIS LESSON**

This lesson uses a real-world situation to introduce students to a constant rate of change and to define the rate of change as slope. Students begin by calculating data points, using a process column, and then determine rates of change between a variety of those points to establish the constant rate of change. The lesson next introduces the algebraic formula for calculating slope and asks students to apply it to the situation. Students then use their new knowledge to calculate slopes of lines containing given pairs of points, including horizontal and vertical lines, and to recognize how slope is represented in various forms of a linear function.

**OBJECTIVES**

Students will

- Translate a real-world scenario into a table.
- Graph data points.
- Calculate the slope of a line.
- Define slope of a line as constant rate of change.

**COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT**

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill. The star symbol (★) at the end of a specific standard indicates that the high school standard is connected to modeling.

**Targeted Standards – (if used in Grade 8)**

- 8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.  
*See questions 1-2, 4-12, 15*

**Reinforced/Applied Standards – (if used in Grade 8)**

- 8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.  
*See question 8*

**Targeted Standards – (if used in Algebra I)**

- F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★  
*See questions 4-6, 10-12, 15*

**Reinforced/Applied Standards – (if used in Algebra I)**

- F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.  
*See question 8*
- A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  
*See questions 1-2, 8, 15a*

## COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction.

NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

**MP.6:** Attend to precision.

*Students use given variables to identify independent and dependent quantities to write a function rule, evaluate function values, scale and label the axes appropriately, plot points, and use mathematical language to interpret the meaning of the ordered pairs and the slope in the context of the situation.*

**MP.8** Look for and express regularity in repeated reasoning.

*In question 12, students use the previous calculations to interpret and understand the process of calculating slope in order to explain how to calculate the average rate of change from the table, graph, and two coordinate points.*

## FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:

- Use dimensional analysis
- Create a graph of discrete data points

## ASSESSMENTS

The following types of formative assessments are embedded in this lesson:

- Students engage in independent practice.
- Students apply knowledge to a new situation.
- Students summarize a process or procedure.

The following assessments are located on our website:

- Rate of Change: Average and Instantaneous – Algebra 1 Free Response Questions
- Rate of Change: Average and Instantaneous – Algebra 1 Multiple Choice Questions

## MATERIALS AND RESOURCES

- Student Activity pages

**TEACHING SUGGESTIONS**

This lesson can be used to introduce the slope, or constant rate of change, of a linear function. Questions 1 – 4 provide a step-by-step discovery process for connecting rate of change to the frequently-used algebraic formula for slope.

Before the lesson begins, divide students into groups of two or three. Complete the first row in the table with the entire class, demonstrating the process to determine Mrs. Smith’s weight for each week. Have students predict what will happen to her weight as the weeks increase if she is able to lose 0.5 kg per week. For question 3, discuss that, during the week, Mrs. Smith’s weight could increase or decrease; however, her cumulative weight loss for the week must be 0.5 kg/week to follow this plan. For question 8, students should use their table to determine the minimum and maximum values to label the graph. The data from the graph should not be connected because the data is discrete. Remind students that they looked at this information in question 3. Question 15 can be done as a continuation of the group activity or as individual practice.

You may wish to support this activity with TI-Nspire™ technology. See *Working with Fractions and Decimals and Calculations Using Special Keys* in the NMSI TI-Nspire Skill Builders.

Suggested question modifications for additional scaffolding include the following:

- 1 Complete several rows of the table prior to giving the Student Activity pages to the student.
- 4 Provide fill-in-the-blank templates to help the student perform the calculations.
- 8 Mark labels and scales on the axes of the graph grid as hints for determining minimum and maximum values.
- 10 Provide a fill-in-the-blank template to help the student perform the calculations.

## NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI’s goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from sixth grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7 contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.



# Slope Investigation

## Answers

1.

Week	Process Column	Weight
0	$120 - 0(.5)$	120 kg
1	$120 - 1(.5)$	119.5 kg
2	$120 - 2(.5)$	119 kg
3	$120 - 3(.5)$	118.5 kg
4	$120 - 4(.5)$	118 kg
5	$120 - 5(.5)$	117.5 kg
6	$120 - 6(.5)$	117 kg
7	$120 - 7(.5)$	116.5 kg
8	$120 - 8(.5)$	116 kg
9	$120 - 9(.5)$	115.5 kg
10	$120 - 10(.5)$	115 kg
n	$120 - n(.5)$	

2.  $w = 120 - 0.5n$

3. No. The domain can only include whole numbers. Since the data is provided for weeks only, it is unknown what happens during the week. She could gain and lose weight within the week as long as her cumulative weight loss for the week is 0.5 kg per week. For this reason, the data is not continuous.

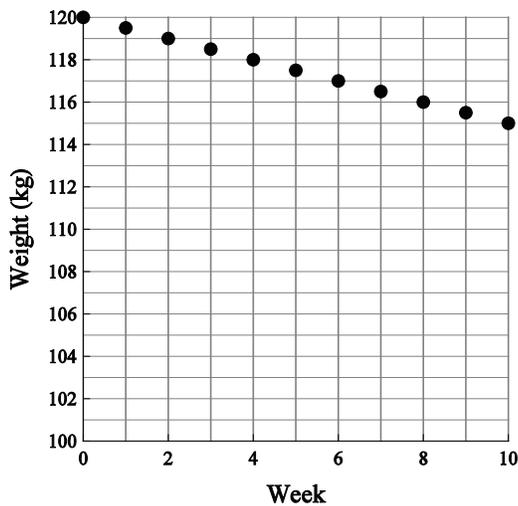
 4. (3, 118.5) and (4, 118). The rate of change is  $\frac{(118.5 - 118)}{(3 - 4)}$ , so the rate of change is  $-0.5 \frac{\text{kg}}{\text{week}}$ .

 5. (3, 118.5) and (5, 117.5). The rate of change is  $\frac{(118.5 - 117.5)}{(3 - 5)}$ , so the rate of change is  $-0.5 \frac{\text{kg}}{\text{week}}$ .

 6. (2, 119) and (8, 116). The rate of change is  $\frac{(119 - 116)}{(2 - 8)}$ , so the rate of change is  $-0.5 \frac{\text{kg}}{\text{week}}$ .

 7. The rate of change is constant,  $-0.5 \frac{\text{kg}}{\text{week}}$ .

 8. *x*-minimum      0      *The smallest number of weeks in the table is 0.*
*x*-maximum      10      *The largest number of weeks in the table is 10, but a larger maximum is acceptable.*
*x*-scale      1      *This scale will match the table.*
*y*-minimum      —      *Answers may widely vary here, from 0 to near 115 kg.*
*y*-maximum      120      *The largest or the starting amount.*
*y*-scale      —      *Answers will vary here depending on the minimum chosen.*



9. The data cannot be connected with a line segment because it is not continuous. Since the data is provided for weeks only, it is unknown what happens during the week. She could gain and lose weight within the week as long as her cumulative weight loss for the week is 0.5 kg per week. For this reason, the data is not continuous.

10. Example:  $(4, 118), (9, 115.5)$  slope  $= \frac{115.5 - 118}{9 - 4} \frac{\text{kg}}{\text{week}} = -0.5 \frac{\text{kg}}{\text{week}}$ .

Check students' work to be sure that they listed the points as ordered pairs, showed determining the slope, and arrived at an answer of  $-0.5 \frac{\text{kg}}{\text{week}}$ .

11. Check students' work to be sure that they listed the points as ordered pairs, showed determining the slope, and arrived at an answer of  $-0.5 \frac{\text{kg}}{\text{week}}$ .

12. Using the table, calculate the difference between any two listed weights and divide by the difference in the corresponding weeks. For any two ordered pairs,  $(x_1, y_1)$  and  $(x_2, y_2)$ , the slope is  $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ . Using the graph, divide the "rise" (vertical change) by the "run" (horizontal change).

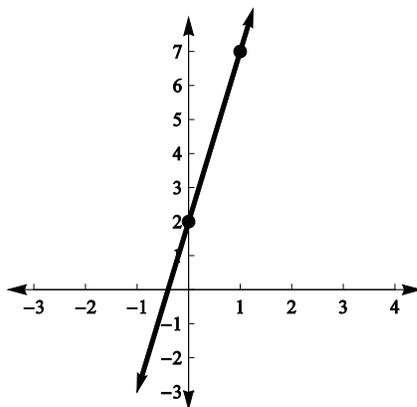
13. At the end of week 20, Mrs. Smith could expect to weigh 242.55 pounds, since

$$\left( 120 - \left( 0.5 \frac{\text{kg}}{\text{week}} \right) (20 \text{ weeks}) \right) \left( \frac{2.205 \text{ lbs}}{1 \text{ kg}} \right) = 242.55 \text{ lbs} .$$

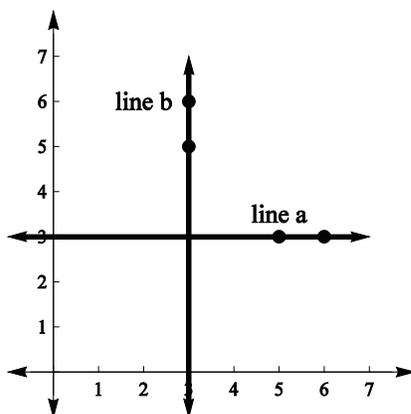
14. Mrs. Smith could expect to weigh 205.065 pounds after 54 weeks, since

$$205.065 \text{ lbs} \left( \frac{1 \text{ kg}}{2.205 \text{ lbs}} \right) = 93 \text{ kg} \text{ and } 93 = 120 - 0.5n; n = 54 .$$

15. a.  $\frac{7-2}{1-0} = 5$



b.  $\frac{3-3}{6-5} = \frac{0}{1}$ ;  $\frac{6-5}{3-3} = \frac{1}{0}$



c.  $\frac{0}{1} = 0$ , this graph is a horizontal line whose slope is zero.  $\frac{1}{0}$  is undefined and the graph is a vertical line. The lines are perpendicular to one another.

d.  $\frac{5-2}{2-0} = \frac{11-5}{6-2} = \frac{11-2}{6-0} = \frac{3}{2}$ , so the points are on the same line.

e.  $\frac{2-0}{3-1} \neq \frac{0-(-5)}{1-(-2)} \neq \frac{2-(-5)}{3-(-2)}$ , so the points are not on the same line.

f. Students may select any two points on the line and calculate the slope. For example,

$$\frac{f(1) - f(0)}{1 - 0} = \frac{5 - 3}{1} = 2$$

g. Students may select any two points on the line and calculate the slope. For example,

$$\frac{g(1) - g(0)}{1 - 0} = \frac{-5 - (-2)}{1} = -3$$

h. Students may select any two points on the line and calculate the slope. For example,

$$\frac{h(3) - h(0)}{3 - 0} = \frac{4 - 2}{3} = \frac{2}{3}$$

i. The slope is the number by which the variable is multiplied.

j. Solve for  $y$  in the first equation.  $y = -\frac{2}{3}x + 2$ , so the slope is  $-\frac{2}{3}$ . Since the second equation is in point-slope form, the slope is displayed in the equation. The slope is  $-\frac{3}{4}$ .

## Slope Investigation

At her annual physical, Mrs. Smith, the health teacher at Smedley Middle School, finds that she is 153 centimeters tall and that she weighs 120 kilograms. After being diagnosed with hypertension (high blood pressure) and high cholesterol, she decides that she will follow a diet planned and supervised by her doctor. On this diet, Mrs. Smith plans to lose 0.5 kg per week.

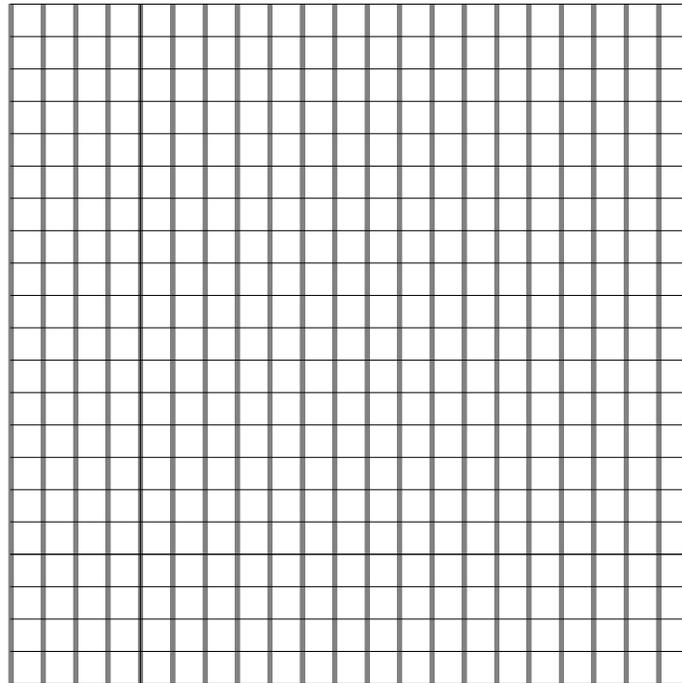
- Complete the table at right to show Mrs. Smith's weight during the first 10 weeks of her diet, if she is able to lose 0.5 kg per week.
- Using the variables  $n$  for week number and  $w$  for weight, write a rule to describe Mrs. Smith's weight after  $n$  weeks.
- Is there information provided for mid-week values? Can Mrs. Smith's weight fluctuate during the week as long as her cumulative weight loss for the week is 0.5 kg? How do these answers affect the domain of the data?

Week	Process Column	Weight
0		120 kg
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
$n$		

- Using the ordered pairs  $(n, w(n))$ , write the ordered pairs that represent her weight for weeks 3 and 4. What is Mrs. Smith's rate of change between weeks 3 and 4? Include units in the answer.
- What is her rate of change in kilograms per week between weeks 3 and 5? Show the points and the method used to calculate the answers.
- What is her rate of change in kilograms per week between weeks 2 and 8? Show the points and the method used to calculate the answers.
- What conclusion can you draw about the rate of change for this data?

8. Determine an appropriate window for the data then graph the data on the grid provided. Label both axes and the graph.

*x*-minimum  
*x*-maximum  
*x*-scale  
*y*-minimum  
*y*-maximum  
*y*-scale



9. Explain why the data cannot be connected with a line segment.



15. Complete the following:
- Using the slope formula  $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ , calculate the slope of the line containing the points  $(0, 2)$  and  $(1, 7)$ . Sketch a graph of these two points and draw a line that contains them.
  - Calculate the slope of the line containing the points  $(5, 3)$  and  $(6, 3)$ . Sketch a graph of these two points and draw a line that contains them, labeling the line  $a$ . Repeat with the points  $(3, 5)$  and  $(3, 6)$ , labeling the line  $b$ .
  - Write a statement about the slopes and graphs in part (b).
  - One method that can be used to show that three or more points lie on the same line is to determine that the slope between pairs of points are the same. Show whether the points  $(0, 2)$ ,  $(2, 5)$  and  $(6, 11)$  do or do not lie on the same line.
  - Show whether the points  $(3, 2)$ ,  $(1, 0)$ , and  $(-2, -5)$  do or do not lie on the same line.

- f. For the function  $f(x) = 2x + 3$ , determine two points on the graph and calculate the slope between the two points. Show your work.
- g. For the function  $g(x) = -3x - 2$ , determine two points on the graph and calculate the slope between the two points. Show your work.
- h. For the function  $h(x) = \frac{2}{3}x + 2$ , determine two points on the graph and calculate the slope between the two points. Show your work.
- i. How is the slope of a function represented in the function rule?
- j. Consider the equations  $2x + 3y = 6$  and  $y = -\frac{3}{4}(x - 2) + 7$ . Explain how you can determine the slope for each equation.

